ACAMAR 10

12-15 May 2024, Guangzhou, China

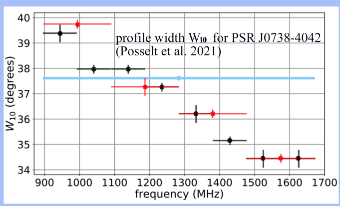
Rotating Vector Model and Radius-to-frequency Mapping in the Presence of a Multipole Magnetic Field

J. L. Qiu, H. Tong, and H. G. Wang

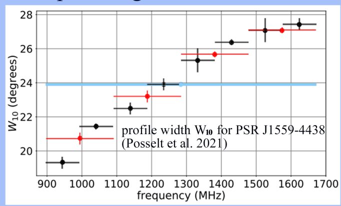
1. Introduction

pulsar's conventional magnetosphere model: a large-scale dipole magnetic field
radio radiation: curvature radiation, narrowband emission

① **RFM** (radius-to-frequency mapping) results from dipole geometry

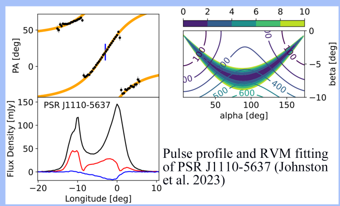


anti-RFM suggests the possible existence of multipole magnetic field



② **Position angle of the linearly polarized radiation component (PA):**

original RVM: a dipole field is assumed, and the PA swing is a typical **S-like swing**



Pulse profile and RVM fitting of PSR J1110-5637 (Johnston et al. 2023)

Magnetar Swift J1818.0−1607: change of PA slope → appearance and disappearance of multipole field with time
FRB (fast radio burst): diversity of period-folded PA features of the bursts (Luo et al. 2020)

observation: anti-RFM & variety of PA
→ multipole magnetic fields may exist in pulsar's magnetosphere
→ correction of RFM and RVM in the presence of multipolar fields

2. Description of the Multipole Magnetic Field

Considering a simple axisymmetric and force-free magnetic field in vacuum, then we get the general expression of the scalar potential

$$\phi(r,\theta) = \sum_l B_l r^{-l-1} P_l(\cos\theta)$$

and magnetic field

$$\vec{B}_l = \frac{B_l}{r^3}(l+1)[P_l(\cos\theta)\hat{r} + \frac{P'_l(\cos\theta)}{l+2}\sin\theta\hat{\theta}]$$

(in the magnetic frame). The field line can be governed by the differential equation

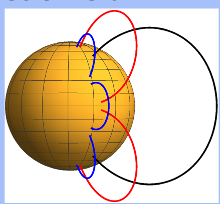


Figure 1. Dipole (black), quadrupole (red), and octopole (blue) magnetic field lines in three dimensions of a neutron star.

3. Relation between Emission Point and Line of Sight

tangent vector of field line $\hat{t} = \frac{\vec{B}}{|\vec{B}|}$ at emission pointis // line of sight of the observer $\hat{l} = \sin\theta_{obs}\cos\phi_{obs}\hat{x} + \sin\theta_{obs}\sin\phi_{obs}\hat{y} + \cos\theta_{obs}\hat{z}$

For a small θ, the relation between the emission point at (r,θ) and the size of the emission cone finally can be simplified as:

$\theta_{obs} = 1.5\theta$ (l=1, dipole magnetic field case)

$\theta_{obs} = 2\theta$ (l=2, quadrupole magnetic field case)

$\theta_{obs} \approx (\frac{l}{2}+1)\theta$ (l-pole magnetic field case)

This qualitatively reveals the frequency evolution behavior of the pulse profile described by RFM phenomenon.

4. RVM in the Presence of the Multipole Magnetic Field

Differential geometry: PA can be calculated by $\tan\psi = \frac{\hat{b}\cdot\hat{\Omega}}{\hat{b}\cdot(\hat{t}\times\hat{\Omega})}$

the vector $\hat{\Omega}$, $\hat{b}$, and $\hat{t}$ are all the same for a dipole or multipole field → the expression for PA will also be the same for a dipole or multipole field.

Spherical trigonometry: PA for a multipole field is

$$\tan(\psi-\psi_0) = \frac{\sin\alpha_m \sin(\phi-\phi_{0,m})}{\cos\alpha_m \sin\zeta - \sin\alpha_m \cos\zeta\cos(\phi-\phi_{0,m})}$$

which is the same as the dipole case

$$\tan(\psi-\psi_0) = \frac{\sin\alpha \sin(\phi-\phi_0)}{\cos\alpha \sin\zeta - \sin\alpha \cos\zeta\cos(\phi-\phi_0)}$$

There are only two constants (α_m and $\phi_{0,m}$) that differ between them.

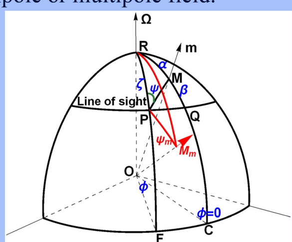


Figure 2. Geometry of the RVM in the presence of the multipole field

5. Radiation Beam Evolution in Coexistence with Dipole and Quadrupole Fields

5.1 Pure Dipole Field

curvature radius: $\rho = \frac{1}{|\vec{\kappa}|} = r\frac{(5+3\cos2\theta)^{\frac{3}{2}}}{3\sqrt{2}\sin\theta(3+\cos2\theta)}$

expression of the dipole field line: $r = r_e\sin^2\theta$

curvature radiation: $\nu = \frac{3\gamma^3 c}{4\pi\rho}$

radiation beam radius: $\rho_{beam} = \theta_{obs} = 1.5\theta = \frac{9}{8}\frac{\rho}{r_e} = \frac{27}{32\pi}\frac{\gamma^3 c}{r_e\nu}$ (θ is assumed as a small angle)

$\rho_{beam} \propto {}^{1}/_{\nu}$ → consistent with the onserved RFM phenomenon

5.2 Pure Quadrupole Field

curvature radius: $\rho = r\frac{[12\cos2\theta + 5(3+\cos4\theta)]^{\frac{3}{2}}}{2\sqrt{2}\sin\theta(39+20\cos2\theta+5\cos4\theta)}$

expression of the field line: $r = r_e\sin^2\theta$

curvature radiation: $\nu = \frac{3\gamma^3 c}{4\pi\rho}$

radiation beam radius: $\rho_{beam} = \theta_{obs} = 2\theta = 4\frac{\rho}{r_e} = \frac{3}{\pi}\frac{\gamma^3 c}{r_e\nu}$ (θ is assumed as a small angle)

$\rho_{beam} \propto {}^{1}/_{\nu}$ → consistent with RFM, but ρ_{beam} is indeed wider than that in dipole case → coexistence of dipole and quadrupole fields should be considered

5.3 An Aligned Dipole and Quadrupole Field

potential: $\Phi_{obs} = \frac{B_{dip}}{2}\frac{R^3}{r^2}\cos\theta + \frac{B_{quad}}{2}\frac{R^4}{r^3}\frac{3\cos^2\theta-1}{2}$ ($B_{dip} = \frac{2B_1}{R^3}$ and $B_{quad} = \frac{3B_1}{R^3}$)

↓ $\vec{B} = -\nabla\Phi$

magnetic field: $\vec{B}_a = (\frac{\cos\theta}{r^3} + b_q\frac{3\cos^2\theta-1}{2r^4})\hat{r} + (\frac{\sin\theta}{2r^3} + b_q\frac{\sin\theta\cos\theta}{r^4})\hat{\theta}$

(dimensionless treatment is used for the retardation height r and the magnetic field strengths ($B_{dip} = 1$ and $B_{quad} = b_q$)

↓

$\rho_{beam} = \frac{4b_q+3r}{2(b_q+r)} = \frac{3\theta}{2} + \frac{b_q}{2(b_q+r)}\theta$

↓ assuming $\nu = \nu_{eq}$ and $\theta = \theta_{eq}$ at $r = b_q$; introducing the dimensionless beam radius: $\rho'_{beam} = {}^{\rho_{beam}}/_{\theta}$

$\rho'_{beam} = \frac{3}{2} + \frac{\nu^2}{2(1+\nu^2)}$

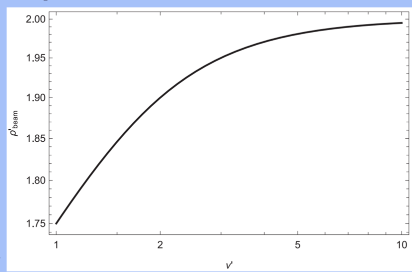


Figure 3. Diagram depicting the trend between the dimensionless radiation beam radius and frequency.

5.4 A Misaligned Dipole and Quadrupole Field

potential: $\Phi_{tot} = \frac{B_{dip}}{2}\frac{R^3}{r^2}\cos\theta + \frac{B_{quad}}{2}\frac{R^4}{r^3}\frac{3\cos^2\theta_q-1}{2}$ (assuming the magnetic axis of dipole field coincides with the rotation axis, and the magnetic inclination angle of quadrupole field is δ)

↓ $\vec{B} = -\nabla\Phi_{tot}$ dimensionless treatment for r and the magnetic field strengths

$\rho_{beam} = \frac{4b_q + 3r - 2b_q\delta\cos\phi}{2(b_q+r)}$

↓

$\rho'_{beam} = \frac{3}{2} + \frac{\nu^2(1-\frac{2\delta\cos\phi}{\theta})}{2(1+\nu^2)}$

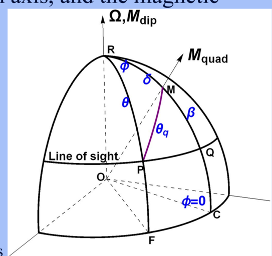


Figure 4. Magnetic field structure with misaligned quadrupole and dipole fields

6. Discussion

① Narrowband emission is comsidered in this paper; Broadband emission model: radiation with a broadband frequency can arise from a narrow range in radius.
② Compared with Yamasaki et al. (2022), we consider the relationship between the line of sight and the emission point in the case of the general multipole field, and use the RVM as a geometric constraint.
③ The emission height can be deduced when the specific radiation and Lorentz factor γ are determined.

7. Comclusion

The modification of RVM and RFM in the presence of multipole magnetic fields are calculated in this research.

A pure dipole field, a pure quadrupole field, a superposition of dipolar and quadrupolar fields are considered to describe pulsar's magnetosphere.

As for RVM in the presence of multipole field, the expression of PA will be the same, with possible changes in the inclination α and phase constant parameters ϕ_0.

For the evolution of radio pulse profile width with frequency, a pure dipole or quadrupole field can explain RFM; when both of them coexist, the trend of profile evolution can explain the anti-RFM phenomenon in some frequency range.